EECS 16B ## 4.14 LECTURE 23

STATE FEEDBACK CONTROL

$\vec{x}(t+1) = A\vec{x}(t) + Bu(t) \qquad u(t) \in \mathbb{R}$

How do we ensure $\vec{x}(t) \to 0$, the equilibrium?

If $|\lambda_i(A)| < 1 \;\forall i$, the system is stable & $\vec{x}(t) \to 0$ w/no input.

If $|\lambda_i(A)| > 1$ for some i, open-loop unstable system. How to stabilize?

CONTROL POLICY

$u(t) = k_1 x_1(t) + k_2 x_2(t) + \dots + k_n x_n(t) = [k_1 \cdots k_n]\,\vec{x}(t) = K\vec{x}(t)$

This creates feedback / a closed-loop control system.

$\vec{x}(t+1) = A\vec{x}(t) + BK\vec{x}(t) = (A+BK)\vec{x}(t)$

Design K such that $|\lambda_i(A+BK)| < 1 \;\forall i$, then we have a stable system.

Is this always possible? Yes, if the system is controllable.

EX $\vec{x}(t+1) = \begin{bmatrix} 0 & 1 \\ \alpha_1 & \alpha_2 \end{bmatrix}\vec{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \qquad A+BK = \begin{bmatrix} 0 & 1 \\ \alpha_1 & \alpha_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ k_1 & k_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \alpha_1 + k_1 & \alpha_2 + k_2 \end{bmatrix}$

$\lambda(A): (-\lambda)(\alpha_2 - \lambda) - \alpha_1 = \lambda^2 - \lambda\alpha_2 - \alpha_1 \longrightarrow \lambda = \frac{\alpha_2 \pm \sqrt{\alpha_2^2 + 4\alpha_1}}{2}$

$\lambda(A+BK): \lambda^2 - (\alpha_2 + k_2)\lambda - (\alpha_1 + k_1)$ To get specific λ values, just match this equation to $(\lambda - \lambda_1)(\lambda - \lambda_2)$!

$\lambda^2 - (\lambda_1 + \lambda_2)\lambda + \lambda_1\lambda_2 \longrightarrow k_1 = -\lambda_1\lambda_2 - \alpha_1 \qquad k_2 = \lambda_1 + \lambda_2 - \alpha_2$

$[B \;\; AB] = \begin{bmatrix} 0 & 1 \\ 1 & \alpha_2 \end{bmatrix} \longrightarrow$ independent, so controllability test works

EX $\vec{x}(t+1) = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}\vec{x}(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) \qquad \lambda(A) = 1, 2$ neither <1, so this is unstable without inputs

$A+BK = \begin{bmatrix} 1+k_1 & 1+k_2 \\ 0 & 2 \end{bmatrix} \qquad \lambda(A+BK) = \underline{k_1+1}, \underline{2} < 1$, so unstable even with inputs

↳ can assign arbitrarily

$[B \;\; AB] = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \longrightarrow$ dependent, so system is uncontrollable

May prove link between stability & controllability in a future lecture.

CONTINUOUS TIME

$\frac{d}{dt}\vec{x}(t) = (A+BK)\vec{x}(t)$ find eigenvalues of $A+BK$ (if controllable) such that $\mathrm{Re}\{\lambda_i(A+BK)\} < 0 \;\forall i$.