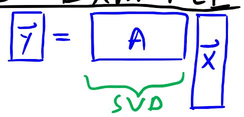
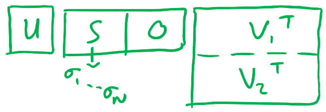
# EECS 16B ## 4.7 LECTURE 21 ##

## MNS EXAMPLE





the smallest norm $\|\vec{x}\|$ is achieved by $\vec{x} = V_1 S^{-1} U^T \vec{y}$.

EX **Case Study:** minimum energy control

friction 0

$u$ = wheel torque

$\frac{d}{dt} p(t) = v(t) \qquad \frac{d}{dt} v(t) = \frac{1}{MR} u(t)$

$$\frac{d}{dt}\begin{bmatrix} p(t) \\ v(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{\lambda_1 = \lambda_2 = 0} \begin{bmatrix} p(t) \\ v(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{RM} \end{bmatrix} u(t)$$

this is continuous, so let's discretize it

$\lambda_1 = \lambda_2 = 0 \longrightarrow A\vec{v} = \lambda\vec{v} = 0 \longrightarrow$ not diagonalizable

integrate to discretize: $dv/dt = \frac{1}{RM} \underbrace{u_d(k)}_{\text{constant!}} \longrightarrow v(t) = v(kT) + \overbrace{(t - kT)}^{T} \frac{1}{RM} u_d(k)$

$dp/dt = \text{⅃}$ for $t < kT + T$ $\begin{cases} p(t) = p(kT) + (t-kT)v(kT) \\ \quad + \frac{1}{2}(t-kT)^2 \frac{1}{RM} u_d(k) \end{cases}$

$$\begin{bmatrix} p_d(k+1) \\ v_d(k+1) \end{bmatrix} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_d(k) \\ v_d(k) \end{bmatrix} + \frac{1}{RM} \begin{bmatrix} \frac{1}{2}T^2 \\ T \end{bmatrix} u_d(k)$$

initially, $p(0) = v(0) = 0$. want to reach $p_t$ & stop ($v_t = 0$)

$$\begin{bmatrix} p_t \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} \vec{b} & A\vec{b} & \dots & A^{l-1}\vec{b} \end{bmatrix}}_{\mathcal{C}_l} \begin{bmatrix} u_d(l-1) \\ \vdots \\ u_d(0) \end{bmatrix}$$

do cols span $\mathbb{R}^2$? check for $l = 2$ $\rightarrow \mathcal{C}_2 = \frac{1}{RM}\begin{bmatrix} \frac{1}{2}T^2 & \frac{3}{2}T^2 \\ T & T \end{bmatrix}$ yes! controllable

## STABILITY

scalar: $x(t+1) = \lambda x(t) + bu(t)$

$x(1) = \underline{\lambda x(0) + bu(0)}$

$x(2) = \lambda x(1) + bu(1) = \lambda^2 x(0) + \lambda b u(0) + bu(1)$

$\vdots$

$$x(t) = \lambda^t x(0) + \underbrace{\begin{bmatrix} b & \lambda b & \cdots & \lambda^{t-1} b \end{bmatrix} \begin{bmatrix} u(t-1) \\ \vdots \\ u(0) \end{bmatrix}}$$

$$\underbrace{\lambda^t x(0) +}_{\text{due to init.}} \underbrace{\sum_{k=0}^{t-1} \lambda^{t-1-k} b u(k)}_{\text{due to input}}$$

this system is stable if state $x(t)$ remains bounded for any $x(0)$ & any bounded input sequence. if any $x(0)$ & BIS such that $|x| \to \infty$, <u>unstable</u>.

CLAIM $|\lambda| > 1 \longrightarrow$ unstable

PROOF $u = 0, \; x(0) \neq 0 \qquad x(t) = \lambda^t x(0)$

$|x(t)| = |\lambda^t| \cdot |x(0)|$

$\hookrightarrow \infty \longrightarrow |x(t)| \to \infty$