EECS 16B ##3.17 LECTURE 17##

So far, we've discussed state space models, controllability, and a little bit of system identification. This leads us into module 3: learning.

System ID: Scalar

$x(t+1) = \lambda x(t) + bu(t)$ $\qquad$ $y(t) = x(t) + e(t)$

$$\underbrace{\begin{bmatrix} x(0) & u(0) \\ \vdots & \vdots \\ x(L-1) & u(L-1) \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} \lambda \\ b \end{bmatrix}}_{\vec{p}} + \underbrace{\begin{bmatrix} e(1) \\ \vdots \\ e(L) \end{bmatrix}}_{\vec{e}} = \underbrace{\begin{bmatrix} y(1) \\ \vdots \\ y(L) \end{bmatrix}}_{\vec{y}} \qquad \hat{\vec{p}} = (D^TD)^{-1}D^T\vec{y}$$

System ID: Vector

$\vec{x}(t+1) = A\vec{x}(t) + B\vec{u}(t) + \boxed{\vec{e}(t)}$ $\longrightarrow$ moved here for clarity

$$\cancel{\begin{bmatrix} \vec{x}(0) & \vec{u}(0) \\ \vdots & \vdots \\ \vec{x}(L-1) & \vec{u}(L-1) \end{bmatrix}} \begin{bmatrix} A \\ B \end{bmatrix}$$

$\vec{x}(0)A \neq A\vec{x}(0)$ $\qquad$ $\dim(\vec{x}) \neq \dim(\vec{u})$

Can't use this!

$\vec{x}(t+1)^T = \vec{x}(t)^T A^T + \vec{u}(t)^T B^T + \vec{e}(t)^T$

$$\underbrace{\begin{bmatrix} \vec{x}(0)^T & \vec{u}(0)^T \\ \vdots & \vdots \\ \vec{x}(L-1)^T & \vec{u}(L-1)^T \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} A^T \\ B^T \end{bmatrix}}_{[\vec{p}_1 \dots \vec{p}_N]} + \underbrace{\begin{bmatrix} \vec{e}(0)^T \\ \vdots \\ \vec{e}(L-1)^T \end{bmatrix}}_{[\vec{e}_1 \dots \vec{e}_N]} = \underbrace{\begin{bmatrix} \vec{x}(1)^T \\ \vdots \\ \vec{x}(L)^T \end{bmatrix}}_{[\vec{y}_1 \dots \vec{y}_N]}$$

Decompose into N separate equations: $D\vec{p}_i + \vec{e}_i = \vec{y}_i$

$\hat{\vec{p}}_i = (D^TD)^{-1}D^T\vec{y}_i$

Singular Value Decomposition

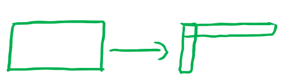
Separates a matrix $A \in \mathbb{R}^{m \times n}$ of rank r into a sum of r rank-1 matrixes of the outer product form $\vec{u}\vec{v}^T$. $A = \sigma_1\vec{u}_1\vec{v}_1^T + \dots + \sigma_r\vec{u}_r\vec{v}_r^T$

Where $\vec{u}_1 \dots \vec{u}_r$ are orthonormal ($\vec{u}_i \perp \vec{u}_j$ and $|\vec{u}_i| = |\vec{u}_j| = 1$), $\vec{u}_i^T\vec{u}_j = 0 \rightarrow i \neq j$

$\vec{v}_1 \dots \vec{v}_r$ are orthonormal, $\sigma_i > 0 \ \forall i$, and $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_r > 0$.

↳ "singular values"

Benefits

- Compression  for images/datasets/anything.
- Feature extraction in large datasets $\longrightarrow$ PCA! After spring break.