EECS 16B ## 3.12 LECTURE 16

CONTROLLABILITY

Recall: a system is controllable if $\text{span}\{\vec{b}, A\vec{b}, \dots, A^{t-1}\vec{b}\} = \mathbb{R}^n$ for some t.
We'll also claim that if $\text{span} \neq \mathbb{R}^n$ for $t=n$, it won't be $\mathbb{R}^n$ for any t.
$\therefore$ $\text{span}\{\vec{b}, A\vec{b}, \dots, A^{n-1}\vec{b}\} = \mathbb{R}^n$ for $t=n$ is a necessary & sufficient condition.

Algorithm: $t=1$ $\quad \text{span}\{\vec{b}\} = \mathbb{R}^n$?
$t=2$ $\quad \text{span}\{\vec{b}, A\vec{b}\} = \mathbb{R}^n$?
$\vdots$

Scenario 1: $\text{span}\{\vec{b}, A\vec{b}, \dots A^{n-1}\vec{b}\} = \mathbb{R}^n$ here, system is controllable & algorithm terminates.

Scenario 2: span keeps growing w/ $t \leq m$, $m < n$. When we increase t to $t = m+1$, span doesn't grow: $A^m\vec{b} = \alpha_0\vec{b} + \alpha_1 A\vec{b} + \dots + \alpha_{m-1}A^{m-1}\vec{b}$
try $t=m+2$ $\longrightarrow$ $A^{m+1}\vec{b} = A(A^m\vec{b}) = \alpha_0 A\vec{b} + \alpha_1 A^2\vec{b} + \dots + \alpha_{m-1}\boxed{A^m\vec{b}}$
here, system is uncontrollable. span remains $m < n$.

EXTENSIONS

1. Multi-input case: similar test! form controllability matrix $C = \underbrace{[B \mid AB \mid \dots \mid A^{n-1}B]}_{A:n\times n,\ B:n\times k,\ \hookrightarrow n\times k}$
check if colspace of C is $\mathbb{R}^n$
2. Continuous-time: $\frac{d}{dt}\vec{x}(t) = A\vec{x}(t) + B\vec{u}(t)$ from C, check if colspace is $\mathbb{R}^n$.

EX see circuit in notes; will make homework easier

SYSTEM IDENTIFICATION

$\vec{x}(t+1) = A\vec{x}(t) + B\vec{u}(t)$ where do A & B come from?
can we learn A & B by observing inputs & resulting states?

Recall least squares. $\vec{y} = D\vec{p} + \vec{e} \longrightarrow \hat{\vec{p}} = \text{proj}_D\,\vec{y} = (D^TD)^{-1}D^T\vec{y}$, which minimizes $\|\vec{e}\|$.
For SysID, start with scalar: $x(t+1) = \lambda x(t) + b\,u(t) + \underline{e(t)}$ error term

$$\begin{bmatrix}\hat{\lambda}\\ \hat{b}\end{bmatrix} = (D^TD)^{-1}D^T\begin{bmatrix}x(1)\\ \vdots \\ x(l)\end{bmatrix} \longleftarrow \underbrace{\begin{bmatrix}x(0) & u(0)\\ \vdots & \vdots \\ x(l-1) & u(l-1)\end{bmatrix}}_{\text{known } D}\underbrace{\begin{bmatrix}\lambda\\ b\end{bmatrix}}_{\vec{p}} + \underbrace{\begin{bmatrix}e(0)\\ \vdots \\ e(l-1)\end{bmatrix}}_{\vec{e}} = \underbrace{\begin{bmatrix}x(1)\\ \vdots \\ x(l)\end{bmatrix}}_{\text{known } \vec{y}}$$

When is D^TD not invertible? Ill-posed setup, such as $x(t) = x^*$ & $u(t) = 0$.
Vector case in notes.