# EECS 16B ## 3.5 LECTURE 14 ##

Equilibrium : $f(\vec{x}^*, \vec{u}^*) = 0$, but in discrete time: $f(\vec{x}^*, \vec{u}^*) = \vec{x}^*$

Linearity : $f(\vec{x}, \vec{u}) = A\vec{x} + B\vec{u}$

$\frac{d}{dt}\vec{x}(t) = A\vec{x}(t) + B\vec{u}(t)$ $\qquad$ $\vec{x}(t+1) = A\vec{x}(t) + B\vec{u}(t)$

these can be algebraically used for equilibrium

if A invertible, $\vec{x}^* = A^{-1}B\vec{u}^*$

## LINEARIZATION IN DISCRETE

$\vec{x}(t+1) = f(\vec{x}(t), \vec{u}(t))$ $\qquad$ $\tilde{x}(t) = \vec{x}(t) - \vec{x}^*$ $\qquad$ $\tilde{u}(t) = \vec{u}(t) - \vec{u}^*$

$x^* = f(\vec{x}^*, \vec{u}^*)$ $\qquad$ so, $\tilde{x}(t+1) = \vec{x}(t+1) - \vec{x}^*$

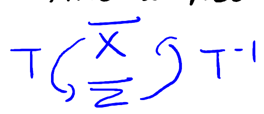
$= f(\vec{x}(t), \vec{u}(t)) - \vec{x}^*$

Taylor: $\cancel{f(\vec{x}^*, \vec{u}^*)} + \underbrace{\nabla_x f(\vec{x}, \vec{u})\Big|_{\vec{x}^*, \vec{u}^*}}_{A} \tilde{x}(t) + \underbrace{\nabla_u f(\vec{x}, \vec{u})\Big|_{\vec{x}^*, \vec{u}^*}}_{B} \tilde{u}(t) \cancel{- \vec{x}^*}$

$\vec{\tilde{x}}(t+1) \approx A\vec{\tilde{x}}(t) + B\vec{\tilde{u}}(t)$

## CHANGE OF BASIS

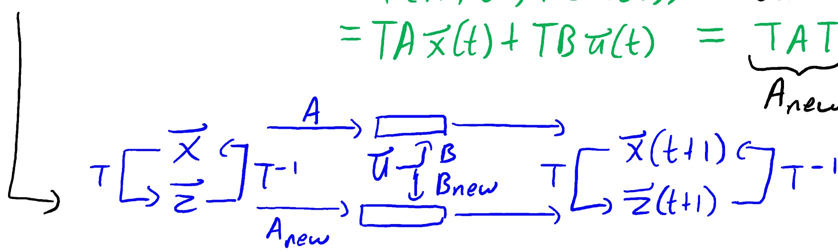
Define a new state vector $\vec{z} = T\vec{x}$ where T is nxn & invertible.




$\vec{z}(t+1) = T\vec{x}(t+1)$

$= T(A\vec{x}(t) + B\vec{u}(t))$ $\quad$ but we don't want $\vec{x}(t)$!

$= TA\vec{x}(t) + TB\vec{u}(t) = \underbrace{TAT^{-1}}_{A_{new}}\vec{z}(t) + \underbrace{TB}_{B_{new}}\vec{u}(t)$



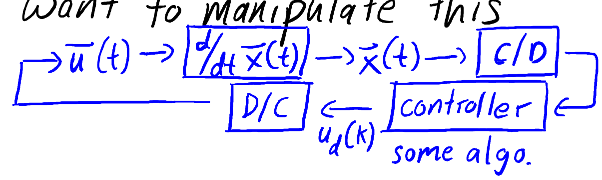

diagonalization is a special case of this!

$A_{new} = V^{-1}AV \rightarrow T = V^{-1}$

## DISCRETIZATION

Can we write continuous models as discrete? $\frac{d}{dt}\vec{x}(t) = A\vec{x}(t) + B\vec{u}(t)$

Our applied inputs are discrete, and we want to manipulate this input to make $\vec{x}(t)$ do our bidding.



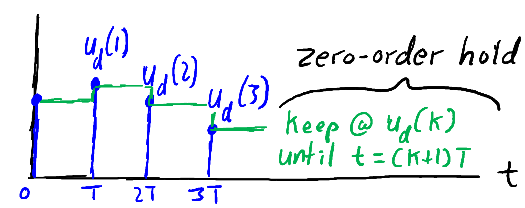

$\boxed{C/D}$: continuous-to-discrete (sampling @ period T)

$\vec{x}(0), \vec{x}(T), \vec{x}(2T) \ldots$

$\vec{x}_d(0), \vec{x}_d(1), \vec{x}_d(2) \ldots$

$\hookrightarrow \vec{x}_d(k) = \vec{x}(kT)$

opposite of $\boxed{D/C}$



$\boxed{D/C} \to \vec{u}(t) \to \boxed{\tfrac{d}{dt}\vec{x}(t)} \to \vec{x}(t) \to \boxed{C/D}$

can we write a discrete-time model for this block ?

Assume scalar: $\frac{d}{dt}x(t) = \lambda x(t) + bu(t)$ $\quad$ $u(t) = u_d(k) \quad kT \le t < kT+T$

take initial time $kT$ w/ $x(kT) = x_d(k)$ & $u_d(k)$ $\quad$ what is $x(kT+T)$ ?

$$x(t) = \underbrace{x_d(k)}_{\text{init}} e^{\lambda(t-kT)} + \int_{\underline{kT}_{\text{init}}}^{t} e^{\lambda(t-\tau)} bu(\tau)\,d\tau \longrightarrow x(kT+T) = e^{\lambda T} x_d(k) + \left(\int_{kT}^{kT+T} e^{\lambda(kT+T-\tau)} d\tau\, b\right) u_d(k)$$

$$x_d(k+1) = \underbrace{e^{\lambda T}}_{\lambda_d} x_d(k) + \underbrace{\left(\int \text{in notes } d\tau\, b\right)}_{b_d} u_d(k)$$