EECS 16B ## 2.27 Lecture 12

Last Class: General Case

$$\frac{d}{dt}\vec{x}(t) = f(\vec{x}(t), \vec{u}(t))$$, where f is a vector function

Linear Case

$$\frac{d}{dt}\vec{x}(t) = A\vec{x}(t) + B\vec{u}(t)$$

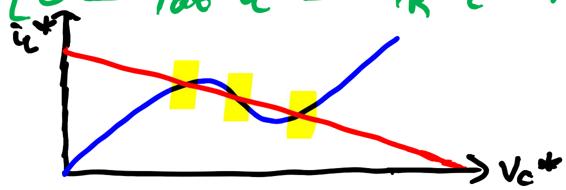
$A \in \mathbb{R}^{n \times n}$ $B \in \mathbb{R}^{n \times m}$
$\vec{x} \in \mathbb{R}^{n}$ $\vec{u} \in \mathbb{R}^{m}$

Equilibrium Points

Set state space derivations to 0 & solve for $i = i^*$ and/or $v = v^*$

DIODE EX from 2.25

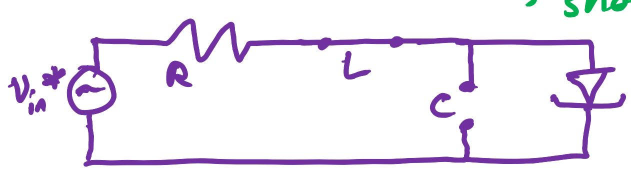
$$\begin{cases} 0 = \frac{d}{dt} v_c^* = \frac{1}{C} i_L^* - \frac{1}{C} g(v_c^*) \\ 0 = \frac{d}{dt} i_L^* = -\frac{1}{R} v_c^* + \frac{1}{R} v_{in}^* - i_L^* \end{cases} \quad \begin{aligned} & i_L^* = g(v_c^*) \\ & i_L^* = -\frac{1}{R}(v_c^* - v_{in}^*) \end{aligned}$$



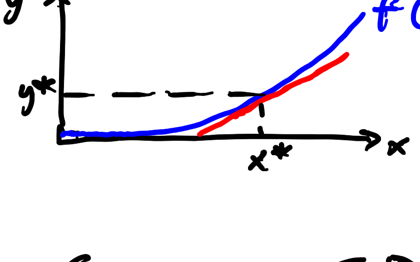
As drawn, there are 3 distinct equilibrium points.

Circuit Analysis

At equilibrium, $V_L = L \frac{d}{dt} i_L = 0$ $i_c = 0$
↳ short ↳ open



Small Signal Linearization



Taylor series @ x^*:

$$y = y^* + \left.\frac{df}{dx}\right|_{x^*}(x - x^*)$$

neglect higher order terms

Vector Case: $\vec{x} \in \mathbb{R}^n$

$$f(\vec{x}) = f(\vec{x}^*) + \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \begin{bmatrix} (x_1 - x_1^*) \\ \vdots \\ (x_n - x_n^*) \end{bmatrix}$$

Jacobian Matrix $\nabla f(\vec{x})\big|_{\vec{x}^*}$