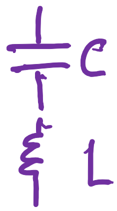
EECS 16B ## 2.25 LECTURE 11

STATE SPACE REPRESENTATION

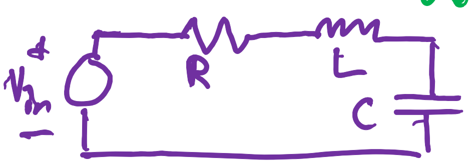
state variables: internal variables that represent systems, typically associated with energy storage / exchange



$C \frac{dV}{dt} = i(t)$

$L \frac{di}{dt} = V(t)$

EX



we want to eliminate everything except V_{in}, i_L, V_c

here, $i_c = i_L$. $V_R = R i_L$. by KVL, $V_L = -V_c - R i_L + V_{in}$

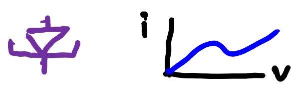
$dV_c/dt = 1/C \; i_L(t)$, $\quad di_L/dt = 1/L \, (-V_c - R i_L + V_{in})$

$$\begin{bmatrix} dV_c/dt \\ di_L/dt \end{bmatrix} = \begin{bmatrix} 0 & 1/C \\ -1/L & -R/L \end{bmatrix} \begin{bmatrix} V_c \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} V_{in}$$

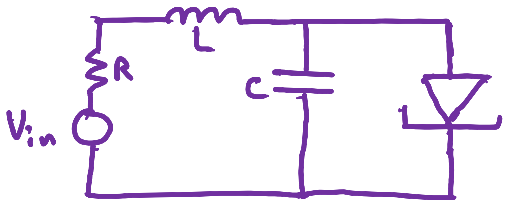
this is familiar...

linear system models can be written in matrix-vector form

EX tunnel diode circuit



$i = g(v)$



diode is static, so state vars are still i_L & V_c

$C \, dV_c/dt = i_c \qquad L \, di/dt = V_L$

KCL: $-i_L + i_c + i_D = 0$, where $i_D = g(v) = g(V_c)$

$C \, dV_c/dt = i_L - g(V_c)$

KVL: same as before. $L \, di/dt = V_{in} - R i_L - V_c$

$dV_c/dt = 1/C \, (i_L - g(V_c))$ $\qquad di/dt = 1/L \, (-V_c - R i_L + V_{in})$

↪ nonlinear

say $h(v) = v(v+1)(v-1) + 4$ cannot linearize

GENERAL FORM

Say we have n state variables $x_1 \dots x_n$ and m inputs $u_1 \dots u_m$. See next page for general form.

$$\frac{dx_1(t)}{dt} = f_1(x_1 \ldots x_n, u_1 \ldots u_m) \rightarrow \frac{dx_n}{dt} = f_n(x_1 \ldots x_n, u_1 \ldots u_m)$$

$$\frac{d}{dt}\vec{x} = f(\vec{x}, \vec{u}) := \begin{bmatrix} f_1(\vec{x}, \vec{u}) \\ \vdots \\ f_n(\vec{x}, \vec{u}) \end{bmatrix}$$

this is linear when $\rightarrow f_i = \underline{a_{i1}}\, x_1 + \underline{a_{i2}}\, x_2 + \ldots + \underline{a_{in}}\, x_n$
$\underline{b_{i1}}\, u_1 + \underline{b_{i2}}\, u_2 + \ldots + \underline{b_{in}}\, u_m$

$\rightarrow f(\vec{x}, \vec{u}) = A\vec{x} + B\vec{u}$ $\rightarrow$ often 1

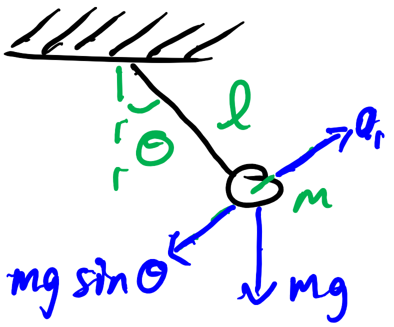
$(n \times n)(n \times 1) \quad (n \times m)(m \times 1)$

Equilibrium States

$d/dt\, \vec{x}(t) = f(\vec{x}(t))$ equilibrium when $f(\vec{x})$ is 0.

if $\vec{x}^*$ is equil. & initial condition, then $\vec{x}(t) = \vec{x}^*$ $t \geq t_0$

EX



$x_1 = \theta$ (radians)

$x_2 = \dfrac{d\theta(t)}{dt}$

velocity $= l\, d\theta/dt$

$a_1 = l\, d^2\theta/dt^2 \leftarrow$ this isn't a state model!

$F = ma$

$ma_1 = -mg \sin\theta$
$- k\, l\, d\theta/dt$

$\rightarrow$ friction

$dx_1/dt = x_2$ $\quad dx_2/dt = -g/l \sin\theta - k/m\, d\theta/dt = -g/l \sin x_1 - k/m\, x_2$

but this is a state model $\quad$ nonlinear