EECS 16B ## 2.20 LECTURE 10

ANALYSIS

$v_{in}(t) = \underbrace{V_{in}}_{\text{real}} \cos\omega t = \mathrm{Re}[V_{in} e^{j\omega t}]$

$v_o(t) = \mathrm{Re}[\underbrace{V_o}_{\text{complex}} e^{j\omega t}]$

Voltage Divider!

$V_o/V_{in} = \underbrace{H(j\omega)}_{\text{transfer}} = \left(\frac{1}{j\omega C}\right)\left(\frac{1}{j\omega C} + R + j\omega L\right)^{-1}$

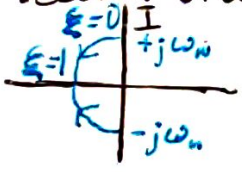
RECALL: TIME DOMAIN ANALYSIS

$A = \begin{bmatrix} 0 & 1/L \\ -1/L & -R/L \end{bmatrix}$ $\quad \det(\lambda I - A) = \lambda^2 + \frac{R}{L}\lambda + \frac{1}{LC}$

$\lambda_{1,2} = -\frac{1}{2}\frac{R}{L} \pm \sqrt{\left(\frac{1}{2}\frac{R}{L}\right)^2 - \frac{1}{LC}}$ may be complex

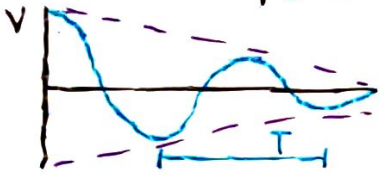
For second-order systems, $\omega_n = \sqrt{\frac{1}{LC}}$ & $\xi = \frac{1}{2} R/\sqrt{L/C}$

↳ greek letter Xi



$\lambda_{1,2} = -\xi\omega_N \pm j\omega_N\sqrt{1-\xi^2}$

time waveform: $e^{\lambda t} = e^{-\xi\omega_N t}\,\mathrm{Re}[e^{j\omega_N\sqrt{1-\xi^2}}]$



$T = 2\pi \cdot (\omega_N\sqrt{1-\xi^2})^{-1}$

FREQUENCY DOMAIN

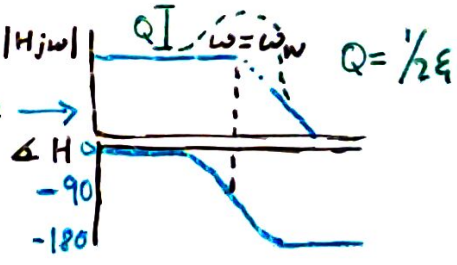
Single frequency ω, phasor analysis w/simplified ω_N & ξ

$$\frac{V_o}{V_{in}} = H(j\omega) = \frac{\omega_N^2}{(j\omega)^2 + (j\omega)2\xi\omega_N + \omega_N^2}$$

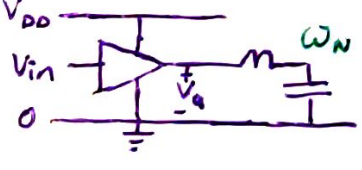
Bode Plot:

1. Low freq.: $\omega << \omega_N$ $\quad H(j\omega) \approx 1$
2. High freq.: $\omega >> \omega_N$ $\quad H(j\omega) \approx -\omega_N^2/\omega^2$

$H(j\omega_N) = \omega_N^2/(j2\xi\omega_N^2) = -j/2\xi$



POSSIBLE APPLICATIONS



$-|V_o| = \frac{1}{2\xi}|V_a|$