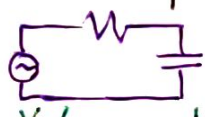


PHASOR SUMMARY

- analysis of one frequency  $\omega = 2\pi f$   $T = 1/f = \text{period}$
- $\frac{d}{dt} \vec{x}(t) = A \vec{x}(t) + \vec{b} u(t)$  for phasors,  $u(t) = U \cos \omega t$
- Euler's Formula  $e^{j\omega t} = \cos \omega t + j \sin \omega t$   $\left\{ \begin{array}{l} \cos \omega t = \text{Re}[e^{j\omega t}] \\ \sin \omega t = \text{Im}[e^{j\omega t}] \end{array} \right.$   
 $\cos \omega t = \frac{1}{2}(e^{j\omega t} + e^{-j\omega t})$   $\sin \omega t = \frac{1}{2j}(e^{j\omega t} - e^{-j\omega t})$
- uniqueness  $x(t) = A \cos(\omega t + \phi) \leftrightarrow A e^{j\phi}$  time  $\leftrightarrow$  phasor
- linearity  $a_1 x_1(t) + a_2 x_2(t) \leftrightarrow a_1 A_1 e^{j\phi_1} + a_2 A_2 e^{j\phi_2}$   $\forall a_1, a_2 \in \mathbb{R}$
- differentiation  $\frac{d}{dt} x(t) \leftrightarrow j\omega A e^{j\phi}$
- application: KVL - linear on branch/phasor voltages  $\begin{array}{l} \text{resistor} \\ \downarrow \\ V = RI \end{array}$   
 $\begin{array}{l} \text{inductor} \\ \downarrow \\ V = j\omega L I \end{array}$  KCL - linear on branch/phasor currents  $\begin{array}{l} \text{capacitor} \\ \downarrow \\ I = j\omega C V \end{array}$   
impedance, which functions like resistor  $\downarrow$  impedance



$v_{in} = V_{in} \cos(\omega t)$   
 $= \text{Re}[V_{in} e^{j\omega t}]$

voltage divider  $V_o = \frac{1/j\omega C}{1/j\omega C + R} V_{in}$   
 $\omega$  impedances!

$V_o/V_{in} = 1/(1 + j\omega RC)$   $\leftarrow$  transfer function: algebraic w/ parameter  $\omega$

FREQUENCY RESPONSE

- Presentation of algebraic function vs. frequency parameter  $\omega$
- log scaling: Bode plots
- $|H(j\omega)|$  vs  $\omega$  log-log scale  $|H|$   $\begin{array}{l} \text{transfer function} \\ H(j\omega) = \end{array}$
- phase of H vs  $\omega$  log scale  
angle of H  $\Theta = \tan^{-1}\left(\frac{\text{Im}(H)}{\text{Re}(H)}\right) = \tan^{-1}(-\omega\tau) = -\tan^{-1}(\omega\tau)$   $\begin{array}{l} \text{phase} \\ \Theta_0 \end{array}$