

PHASORS

Ubiquitous method for understanding response to sine inputs.

- Calculating the particular response given  $\rightarrow$

EX



$$dV_o/dt = -\frac{1}{RC} V_o + \frac{1}{RC} V_{in} \quad \lambda = -1/RC$$

easier calc:  $V_{in} = V_{in} e^{st} \rightarrow v_o = V_o e^{st}$   
 which led to  $sV_o e^{st} = -\frac{1}{RC} e^{st} (V_o - V_{in}) \quad V_o = \frac{V_{in}}{1+sRC}$   
 back to time response:  $v_o = V_o e^{st} + A e^{st}$

So what's a phasor?  $x(t) = A \cos(\omega t + \phi) = \text{Re}(A e^{j\phi} e^{j\omega t})$

A complex number w/amplitude & phase info  $\rightarrow$

Re(): take the real part

PROPERTIES

Uniqueness:  $x(t) = A \cos(\omega t + \phi) \leftrightarrow A e^{j\phi}$   
 time wave form phasor

consider  $x_1(t) = \text{Re}(A_1 e^{j\omega t})$  &  $x_2(t) = \text{Re}(A_2 e^{j\omega t})$

$x_1(t) = x_2(t) \iff A_1 = A_2$

$\text{Re}(A_1 e^{j\omega t}) = \text{Re}(A_2 e^{j\omega t}) \quad \checkmark$

$x_1(0) = \text{Re}(A_1) \quad x_2(0) = \text{Re}(A_2)$

$x_1(\frac{\pi}{2} \frac{1}{\omega}) = \text{Re}(A_1 e^{j\pi/2}) \quad x_2(\frac{\pi}{2} \frac{1}{\omega}) = \text{Re}(A_2 e^{j\pi/2})$

$e^{j\pi/2} = \cos \frac{\pi}{2} + j \sin \frac{\pi}{2} = j \quad \text{Re}(A_1 j) = \text{Re}(A_2 j) \text{ etc.}$

Linearity:  $a_1 x_1(t) + a_2 x_2(t) \leftrightarrow a_1 A_1 + a_2 A_2$

Differentiation:  $d/dt x(t) = \text{Re}(j\omega A e^{j\omega t}) \quad d/dt x(t) \leftrightarrow j\omega A$

IF

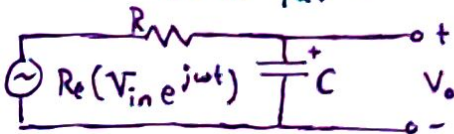
ONLY IF

APPLICATION

$\text{---} \text{||} \text{---} \quad i = C dv/dt \leftrightarrow I = j\omega CV$

$\text{---} \text{---} \text{---} \quad v = L di/dt \leftrightarrow V = j\omega LI$

EX



$V_o = \frac{1}{1+j\omega CR} V_{in}$   
 $v_o(t) = \text{Re}(V_o e^{j\omega t})$

