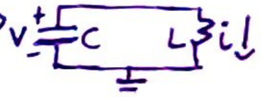
EECS 16B #2.11 LECTURE 7

INDUCTOR CIRCUITS

EX1 We left off with $\frac{d}{dt} \cdot i = -\frac{R}{L} \cdot i + \frac{1}{L} \cdot V_{in}$ $\quad \lambda = -\frac{R}{L}$

Inductor time constant $\tau = \frac{L}{R} = \frac{flux/i}{V/i} = \frac{Vs}{V} =$ seconds

EX2  $C\, \frac{d}{dt} V + i = 0$ $\quad \frac{d}{dt}\begin{bmatrix} V \\ i \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{C} \\ \frac{1}{L} & 0 \end{bmatrix}\begin{bmatrix} V \\ i \end{bmatrix}, \begin{bmatrix} v \\ i \end{bmatrix}$ t=0 check

$L\, \frac{d}{dt}\, i = V$

$L = 1H \quad C = 1F \quad \frac{d}{dt}\vec{x} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\vec{x} \quad \lambda^2 + 1 = 0 \quad \lambda^2 = -1 \quad \lambda = \pm\sqrt{-1}$

$\sqrt{-1} = j \quad \lambda_1 = j \quad \lambda_2 = -j \quad \vec{v}_1 \to \begin{bmatrix} j & 1 \\ -1 & j \end{bmatrix} \to \begin{bmatrix} 1 \\ -j \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} 1 \\ j \end{bmatrix}$

$\lambda_2 = \overline{\lambda_1} \quad \vec{v}_2 = \overline{\vec{v}_1}$ "conjugate" eigenstuff seems conjugate pairs

PRODUCT RULE FOR CONJUGATION

$z_1 \cdot z_2 = |z_1| e^{j\theta_1} \cdot |z_2| e^{j\theta_2} = |z_1||z_2| e^{j(\theta_1+\theta_2)}$

$\overline{z_1} \cdot \overline{z_2} = |z_1| e^{-j\theta_1} \cdot |z_2| e^{-j\theta_2} = |z_1||z_2| e^{-j(\theta_1+\theta_2)} = \overline{z_1 \cdot z_2}$

EX2 $p(\lambda) = 0$ has all real coeff., but λ is complex

$\lambda^n + a_{n-1}\lambda^{n-1} + . + a_1\lambda + a_0 = 0$

$\overline{p(\lambda)} = \overline{\lambda^n} + a_{n-1}\overline{\lambda^{n-1}} + . + a_1\overline{\lambda} + a_0 = 0 = (\overline{\lambda})^n + a_{n-1}(\overline{\lambda})^{n-1} + . + a_1\overline{\lambda} + a_0$

If complex λ_1 is root of $p(\lambda)$, so is $\overline{\lambda}_1$. $\quad \vec{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} v \\ i \end{bmatrix}$

$\vec{x}(0) = \frac{1}{2}\begin{bmatrix} 1 \\ -j \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 1 \\ j \end{bmatrix} \quad \vec{x}(t) = \frac{1}{2}\begin{bmatrix} 1 \\ -j \end{bmatrix} e^{jt} + \frac{1}{2}\begin{bmatrix} 1 \\ j \end{bmatrix} e^{-jt}$

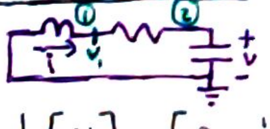
We know $\cos(\theta) = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$ & $\sin(\theta) = \frac{1}{2j}(e^{j\theta} - e^{-j\theta})$ &

$e^{j\theta} = \cos\theta + j\sin\theta$ $\quad -j^2 = 1$

$v(t) = \frac{1}{2}e^{jt} + \frac{1}{2}e^{-jt} = \cos(t) \quad i(t) = \frac{1}{2j}e^{jt} - \frac{1}{2j}e^{-jt} = \sin(t) \cdot -j^2$

$v^2 + i^2 = 1 \quad cv^2 + Li^2 = 2 \cdot \text{energy}$

LRC CIRCUITS



① $-i + \frac{V_1 - V}{R} = 0$ ③ $L\, \frac{d}{dt}\, i = -v_1 \longrightarrow L\, \frac{d}{dt}\, i = -Ri - v$

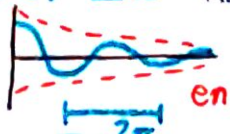
② $\frac{V - V_1}{R} + C\, \frac{d}{dt} V = 0 \longrightarrow C\, \frac{d}{dt}\, v = i \quad \lambda^2 - \frac{R}{L}\lambda + \frac{1}{LC}$

$\frac{d}{dt}\begin{bmatrix} v \\ i \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix}\begin{bmatrix} v \\ i \end{bmatrix} \quad \lambda_{1,2} = -\frac{1}{2}\frac{R}{L} \pm \sqrt{\left(\frac{1}{2}\frac{R}{L}\right)^2 - \frac{1}{LC}}$

$0 \leq \left(\frac{1}{2}\frac{R}{L}\right)^2 \leq \frac{1}{LC} : \lambda_{1,2} = \underbrace{-\frac{1}{2}\frac{R}{L}}_{\lambda_r} \pm j \underbrace{\sqrt{\frac{1}{LC} - \left(\frac{1}{2}\frac{R}{L}\right)^2}}_{\lambda_i}$

$e^{(\lambda_r + j\lambda_i)t} = e^{\lambda_r t} e^{j\lambda_i t}$

$= e^{\lambda_r t}[\cos(\lambda_i t) + j\sin(\lambda_i t)]$



envelope $\cdot\ e^{\lambda_r t} = e^{-\frac{R}{2L}t}$

$T = \frac{2\pi}{\lambda_i}$