EECS 16B ## 2.6 LECTURE 6

LAST CLASS

$d/dt\ \vec{x} = A\vec{x} + \vec{b}u \qquad \vec{x} = \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} \qquad \vec{x}(0)$ is specified

Represent $\vec{x}$ in terms of eigenvectors of A. $\vec{x} = \vec{v}_1\tilde{x}_1 + \vec{v}_2\tilde{x}_2$

(note that v_1, v_2 are voltages, while $\vec{v}_1, \vec{v}_2$ are eigenvectors corresponding to λ_1 & λ_2). $\vec{x} = [\vec{v}_1 | \vec{v}_2]\vec{\tilde{x}} \qquad [\vec{v}_1 | \vec{v}_2] = V$

$\lambda_1 \neq \lambda_2$, so V^{-1} exists.

$V^{-1}AV\vec{\tilde{x}} = \Lambda\vec{\tilde{x}}$

$$\begin{array}{ccc} \vec{x} & \xrightarrow{A} & A\vec{x} \\ {\scriptstyle V^{-1}}\downarrow & & \uparrow{\scriptstyle V} \\ \vec{\tilde{x}} & \xrightarrow{\Lambda} & \Lambda\vec{\tilde{x}} \end{array}$$

$AV = V\Lambda$

$\Lambda = V^{-1}AV$

$\vec{\tilde{b}} = V^{-1}\vec{b}$

SOLUTION PLAN

1. Compute $\lambda_1, \lambda_2, \vec{v}_1, \vec{v}_2$
2. Use $\vec{\tilde{x}} = V^{-1}\vec{x}$; $\vec{\tilde{x}}(0) = V^{-1}\vec{x}(0)$
3. Solve $d/dt\ \vec{\tilde{x}} = \Lambda\vec{\tilde{x}} + V^{-1}\vec{b}u$
4. Recover using $\vec{x} = V\vec{\tilde{x}}$

EX $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 3 \\ 0 \end{bmatrix} \qquad (-5-\lambda)(-2-\lambda) - 4 \qquad \lambda^2 + 7\lambda + 6 = 0$

$(\lambda+5)(\lambda+2) - 4 \qquad \lambda_1 = -1 \quad \lambda_2 = -6$

$\lambda_1 I - A = \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix} \to \begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \vec{v}_1 \qquad \vec{v}_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \qquad V = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

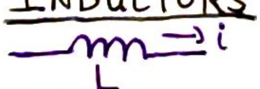
$V^{-1} = \begin{bmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{bmatrix} \qquad d/dt\ \vec{\tilde{x}} = \begin{bmatrix} -1 & 0 \\ 0 & -6 \end{bmatrix}\vec{\tilde{x}} + V^{-1}\vec{b}u \qquad V^{-1}\vec{b} = \begin{bmatrix} 3/5 \\ 6/5 \end{bmatrix}$

$$\vec{\tilde{x}}(t) = \begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{bmatrix}\vec{\tilde{x}}(0) + \int_0^t \begin{bmatrix} e^{\lambda_1(t-\tau)} & 0 \\ 0 & e^{\lambda_2(t-\tau)} \end{bmatrix}\vec{\tilde{b}} \cdot u(\tau)$$

$\tilde{x}_1(t) = e^{\lambda_1 t}\tilde{x}_1(0) + \int_0^t e^{\lambda(t-\tau)}\tilde{b}_1 u(\tau)$ similarly for $\tilde{x}_2(t)$

$\vec{x}(t) = [\vec{v}_1 | \vec{v}_2]\vec{\tilde{x}}(t)$

INDUCTORS



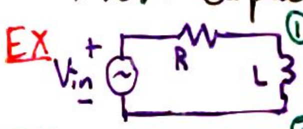
$+ \ \lambda \ -$ (flux) $\qquad d/dt\ \lambda = v_L \qquad i = \lambda/L$

$E = \frac{1}{2}Li^2$

L: Henry

From capacitors, $q \to \lambda$, $i \to v$, $v \to i$ $\qquad \lambda$: Webers = Volt-sec

EX



① $\to \frac{v - v_{in}}{R} + i_L = 0$ ② $L\ d/dt\ i = v \leftarrow$ extra equation

$d/dt\ i = (-Ri + v_{in})/L$

EX

① $\to C\ d/dt\ v = -i$

② $L\ d/dt\ i = v$

$d/dt\begin{bmatrix} v \\ i \end{bmatrix} = \begin{bmatrix} 0 & -1/C \\ 1/L & 0 \end{bmatrix}\begin{bmatrix} v \\ i \end{bmatrix}$ Complex eigenvals incoming :)