EECS 16B   ## 2.4 LECTURE 5

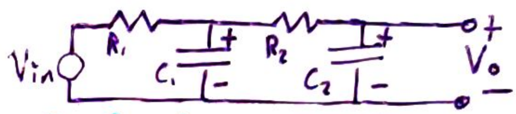
$v_{in}(t) = V_{in} \cdot \cos(\omega t)$ ω = freq. in rad/s $= 2\pi f$ f in Hertz

Guess that $V_o(t) = A\cos(\omega t + \phi)$ $V_o(t) = \dfrac{V_{in} \cdot \cos(\omega t + \phi)}{\sqrt{\omega^2 (RC)^2 + 1}} + Be^{-\frac{t - t_0}{RC}}$

$\phi = -\tan^{-1}(\omega RC)$

More elements, better audio filter?

Second Order Circuit! $\longrightarrow$



$C_1 = C_2 = 1\,\mu F$ $R_1 = 1/3\, M\Omega$ $R_2 = 1/2\, M\Omega$

KCL

$$C_1 \tfrac{d}{dt} V_1 + \frac{V_1 - V_{in}}{R_1} + \frac{V_1 - V_2}{R_2} = 0 = C_2 \tfrac{d}{dt} V_2 + \frac{V_2 - V_1}{R_2}$$

$$\tfrac{d}{dt} V_1 = -V_1 \left(\tfrac{1}{R_1} + \tfrac{1}{R_2}\right)\tfrac{1}{C_1} + V_2 \tfrac{1}{R_2 C_1} + V_{in} \tfrac{1}{R_1 C_1} = -5v_1 + 2v_2 + 3v_{in}(t)$$

$$\tfrac{d}{dt} V_2 = V_1 \tfrac{1}{R_2 C_2} - V_2 \tfrac{1}{R_2 C_2} = 2v_1 - 2v_2$$

VECTOR DIFFERENTIAL / STATE-SPACE EQUATION

$$\frac{d}{dt}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \end{bmatrix} v_{in}(t)$$

$\tfrac{d}{dt}\vec{x} = A\vec{x} + \vec{b}\, u(t)$ Guess: $\tfrac{d}{dt}\vec{x} = A\vec{x} \rightarrow \vec{x}(t) = \vec{v}e^{\lambda t}$ $\rightarrow A = \lambda$

$A\vec{v}_1 = \lambda\vec{v}_1$ $A\vec{v}_2 = \lambda\vec{v}_2$ Presume $\vec{v}_1$ & $\vec{v}_2$ linearly independent

$A[\vec{v}_1 | \vec{v}_2] = [\vec{v}_1 | \vec{v}_2]\Lambda$, where $\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$ $[\vec{v}_1 | \vec{v}_2] = V$

$A = V \Lambda V^{-1} \leftarrow$ eigenvalue/eigenvector decomposition of A

General $\vec{x}(0) = \tilde{x}_1 \vec{v}_1 + \tilde{x}_2 \vec{v}_2$ $\begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} = V^{-1}\vec{x}(0)$

$\vec{x}(t) = \vec{v}_1 e^{\lambda_1 t}\tilde{x}_1(0) + \vec{v}_2 e^{\lambda_2 t}\tilde{x}_2(0)$ $\vec{\tilde{x}} = V^{-1}\vec{x}$

$\tfrac{d}{dt}\vec{\tilde{x}} = V^{-1}(A\vec{x} + \vec{b}u) = V^{-1}AV\vec{\tilde{x}} + V^{-1}\vec{b}u$

$\tfrac{d}{dt}\vec{\tilde{x}} = \Lambda\vec{\tilde{x}} + \vec{\tilde{b}}u$, where $\vec{\tilde{b}} = V^{-1}\vec{b}$