

Continuing from last time:

How much of  $\vec{r}$  is explained by  $\vec{s}_2$ ?  $\hat{\alpha}_2 = \frac{\langle \vec{r}, \vec{s}_2 \rangle}{\|\vec{s}_2\|^2}$

$\vec{e} = \vec{r} - \hat{\alpha}_2 \vec{s}_2$  "error | residual"

Consider  $\langle \vec{e}, \vec{s}_1 \rangle$   $\langle \vec{e}, \vec{s}_{10000} \rangle$

$$\begin{aligned} \langle \vec{e}, \vec{s}_2 \rangle &= \langle \vec{r} - \hat{\alpha}_2 \vec{s}_2, \vec{s}_2 \rangle = \langle \vec{r}, \vec{s}_2 \rangle - \hat{\alpha}_2 \langle \vec{s}_2, \vec{s}_2 \rangle \\ &= \langle \vec{r}, \vec{s}_2 \rangle - \frac{\langle \vec{r}, \vec{s}_2 \rangle}{\|\vec{s}_2\|^2} \cdot \|\vec{s}_2\|^2 = \boxed{0} \end{aligned}$$

Say  $\vec{s}_4$  is the next max  $\vec{r} = \alpha_2 \vec{s}_2 + \alpha_4 \vec{s}_4 +$

Project  $\vec{r}$  onto  $\text{col}\{\vec{s}_2, \vec{s}_4\}$

$S \rightarrow S \begin{bmatrix} \alpha_2 \\ \alpha_4 \end{bmatrix} \approx \vec{r}$  least squares!

$$S \hat{\alpha} = \vec{r} \longrightarrow \hat{\alpha} = (S^T S)^{-1} S^T \vec{r}$$

$$S = [\vec{s}_2 \ \vec{s}_4]$$

When do you stop?

This is a design decision. Depending on application:

- you may know sparsity level (how many devices)
- you may know the noise (stop when error  $\approx$  noise)