# EECS 16A # 11.26 LECTURE 25 ##

For $A^TA$ to be invertible, $\text{Null}(A^TA)$ must be trivial. We proved last time that $\text{Null}(A^TA) = \text{Null}(A)$, which is nontrivial if columns of $A$ are linearly dependent.

REMINDER: $(AB)^T = B^TA^T$ $\quad A: n \times m \quad A^T: m \times n \quad$ cannot
$B: m \times k \quad B^T: k \times m \quad$ multiply $A^TB^T$

## ORTHOGONAL MATCHING PURSUIT

In a smart home, how do tell which devices are talking?

Gold Codes: $\vec{s}_A, \vec{s}_B$ $\quad \pm 1 \quad N = 1023$

$\langle \vec{s}_A, \vec{s}_B \rangle \approx 0 \qquad \langle \vec{s}_A, \vec{s}_A \rangle = N$

$\vec{r} = \alpha_1 \vec{s}_1 + \alpha_2 \vec{s}_2 + \quad \alpha_{10000} \vec{s}_{10000}$

$$\begin{bmatrix} \vec{s}_1 & \vec{s}_2 & \cdots & \vec{s}_{10000} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_{10000} \end{bmatrix} = \underbrace{\begin{bmatrix} \; \end{bmatrix}}_{1023} \vec{r}$$

$\text{len}(\vec{s}_n) = 1023$

Gaussian Elimination AND Least Squares fail here!

To solve using OMP, be greedy.

$\vec{r} = \vec{s}_A + 2\vec{s}_B$ ... consider $\langle \vec{r}, \vec{s}_n \rangle$. most should be small.

$\longrightarrow$ only expect $\langle \vec{r}, \vec{s}_A \rangle$ & $\langle \vec{r}, \vec{s}_B \rangle$ as large

1. Consider all inner products.
2. Find the maximum inner product $\vec{s}_{max}$
   Guess: $\vec{s}_{max}$ is transmitting.
3. Project $\vec{r}$ onto $\vec{s}_{max}$: $\text{proj}_{\vec{s}_{max}}(\vec{r}) \longrightarrow \vec{r}_{new} = \vec{r} - \text{proj}_{\vec{s}_{max}}(\vec{r})$
4. Repeat with $\vec{r}_{new}$ as $\vec{r}$.

Peel one component at a time!