EECS 16A ## 11.21 LECTURE 24

The orthogonal projection of $\vec{b}$ onto col(A) minimizes $\|A\vec{x}-\vec{b}\|^2$

Best estimate $\hat{\vec{x}} = (A^TA)^{-1} \cdot A^T \cdot \vec{b}$

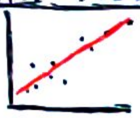
$A\hat{\vec{x}} = A(A^TA)^{-1} \cdot A^T \cdot \vec{b} \approx \vec{b}$

ex. $A = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\vec{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 2 \\ 1 \end{bmatrix} x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Gaussian Elimination yields no solution

$A^T = [2 \ \ 1]$ $A^TA = 5$ $(A^TA)^{-1} = 1/5$ $\hat{\vec{x}} = \frac{1}{5} \cdot 3 = 3/5$

LINEAR REGRESSION



$y = mx + c$

we have many x, y

$$\begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} \begin{bmatrix} m \\ c \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

We can use least squares to find the <u>line of best fit.</u>

Least Squares can fail if A^TA is non-invertible.

THM $\text{Null}(A^TA) = \text{Null}(A) \longrightarrow$ nontrivial, and cols of A are lin dependent

1. If $\vec{u} \in N(A)$, then $\vec{u} \in N(A^TA)$
2. If $\vec{w} \in N(A^TA)$, then $\vec{w} \in N(A)$

PROOF 1. $A\vec{u} = 0$ $(A^TA)\vec{u} = A^T(A\vec{u}) = A^T \cdot 0 = 0 \implies \vec{u} \in N(A^TA)$

2. $A^TA\vec{w} = 0$ $A\vec{w} = \vec{q}$

$\hookrightarrow 0 \to \langle \vec{q}, \vec{q} \rangle = (A\vec{w})^T(A\vec{w}) = \vec{w}^TA^TA\vec{w} = \vec{w}^T \cdot 0$

$= 0 \to \|\vec{q}\| = 0 \to \vec{q} = 0$

$\implies \vec{w} \in N(A)$