EECS 16A ## 11.12 LECTURE 21 ##

MODULE 3: MACHINE LEARNING

This module builds upon the first 2 modules:
- M1: systems & modeling
- M2: analysis & design

We will build a GPS Localization System.

MACHINE LEARNING

Classification : what is this a picture of?
Estimation : estimate parameter from data
Prediction : what path will a pedestrian take?
Clustering : take eecs 16b

TECHNIQUES

Modeling, Optimization of error metric

DESIGN PROBLEM

GPS System : 24 satellites

Need to know: distance between satellites,
distance between you & satellites,
how many satellites are enough,
locations of satellites, information
processing to compute distances, noise,
which satellite am I talking to

what we'll talk about in eecs 16a

INNER PRODUCT

Let $\vec{v}, \vec{w}$ $\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$

Inner Product $\underbrace{\langle \vec{v}, \vec{w} \rangle}_{\text{notation}} = \vec{v}^T \cdot \vec{w}$ $\begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$

Hint: this is a dot product

DEF Inner Product/Correlation of $\vec{v}$ & $\vec{w}$ is $\langle \vec{v}, \vec{w} \rangle = \vec{v} \cdot \vec{w}$

Magnitude/Norm of $\vec{v}$ is $\langle \vec{v}, \vec{v} \rangle = \vec{v} \cdot \vec{v} = \sum_{i=1}^{n} v_i^2$

General 2D Vectors

$$\vec{v} = \alpha \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \qquad \vec{\omega} = \beta \begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix}$$

$$\begin{aligned} \langle v, \omega \rangle &= \alpha\beta(\cos\theta\cos\phi + \sin\theta\sin\phi) \\ &= \alpha\beta\cos(\theta-\phi) \\ &= \|v\| \cdot \|\omega\| \cdot \cos(\theta-\phi) \end{aligned}$$

Inner Product is maximized when $\theta = \phi$

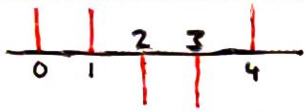
if $\theta - \phi = 90°$, $\cos(\theta-\phi) = 0 = \langle v, \omega \rangle$ "orthogonal"

Cauchy-Schwarz Inequality

$$\langle v, \omega \rangle \leq \|v\| \cdot \|\omega\|$$

Satellite Classification

(A) $\vec{s}_A = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$



(B) $\vec{s}_B =$

These signatures are called "Gold Codes," and are generally much longer. In lab, our codes will be 1024 elements long.

Unfortunately, we usually receive $\vec{s} + \vec{n} = \vec{r}$

$\vec{r} = \begin{bmatrix} 0.9 \\ 1.1 \\ -1.2 \\ 0 \end{bmatrix}$ how do we classify with noise?

Which signature is $\vec{r}$ closest to?

Choose error metric $\vec{e}_x = \vec{r} - \vec{s}_x$

Find satellite x for minimum $\|\vec{e}_x\|^2$

Minimize $\|\vec{e}_x\|^2$ over all satellites x.

$$\begin{aligned} \|\vec{e}_x\|^2 &= \langle \vec{e}_x, \vec{e}_x \rangle = \vec{e}_x^T \cdot \vec{e}_x = (\vec{r}-\vec{s}_x)^T(\vec{r}-\vec{s}_x) = (\vec{r}^T - \vec{s}_x^T)(\vec{r}-\vec{s}_x) \\ &= \vec{r}^T\vec{r} + \vec{s}_x^T\vec{s}_x - \vec{s}_x^T\vec{r} - \vec{r}^T\vec{s}_x = \underbrace{\|\vec{r}\|^2}_{\text{fixed}} + \underbrace{\|\vec{s}_x\|^2}_{\text{fix!}} - 2\langle \vec{r}, \vec{s}_x \rangle \end{aligned}$$

Minimize $-2\langle \vec{r}, \vec{s}_x \rangle \longrightarrow$ Maximize $\langle \vec{r}, \vec{s}_x \rangle$

$[\langle \vec{r}, \vec{s}_x \rangle$ for x in $s] \leftarrow$ return index of maximum