EECS 16A

# ## 9.24 LECTURE 8 ##

Columnspace = Span = Range

## SUBSPACE

Let $(\mathbb{V}, \mathbb{F})$ be a vector space

$(\mathbb{W}, \mathbb{F})$ is a subspace if $\mathbb{W}$ is a subset of $\mathbb{V}$ and if $(\mathbb{W}, \mathbb{F})$ is a vector space

eg. span$\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\end{bmatrix}\right\}$ is a subspace of $\mathbb{R}^3$.

eg. $\mathbb{R}^2$ is not a subspace of $\mathbb{R}^3$.

eg span$\left\{\begin{bmatrix}3\\4\end{bmatrix}\right\}$ is a subspace of $\mathbb{R}^2$.

eg. $\{\vec{0}\}$ is a subspace of $\mathbb{R}^2$.

## NULLSPACE

Solutions to $A\vec{x} = \vec{0}$

**Inv. of 2x2 Matrix**

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

(continued from invertibility section)

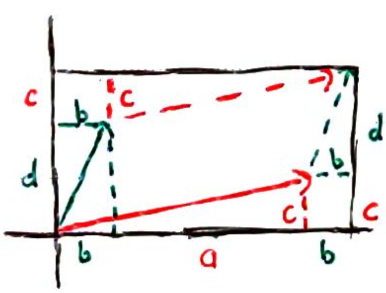
## COLUMNSPACE

Span of independent columns of A

Dimension = # of elements in Basis

**DEF** Rank of a matrix = number of independent columns

Matrix A is invertible if
- A has lin-indep columns
- $A\vec{x} = \vec{b}$ has a unique solution
- Nullspace = $\{\vec{0}\}$ "is trivial"
- Determinant $\neq 0$




$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Big Rectangle : $(a+b)(c+d)$

$= bc + bc$

$= \frac{1}{2}ac + \frac{1}{2}ac$

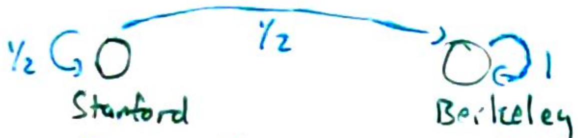
$= \frac{1}{2}bd + \frac{1}{2}bd$

Area $= \square - \ldots$

$= \underline{ad - bc}$

↳ Determinant

# Tale of Two Websites (Page Rank)




$$Q = \begin{bmatrix} 1/2 & 0 \\ 1/2 & 1 \end{bmatrix}$$

$$\vec{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} x_S \\ x_B \end{bmatrix}$$

$$\vec{x}(1) = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

$$\vec{x}(2) = \begin{bmatrix} 1/4 \\ 3/4 \end{bmatrix}$$

$$\vec{x}(t) = \begin{bmatrix} (1/2)^t \\ 1-(1/2)^t \end{bmatrix}$$

$$\vec{x}(\infty) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

If $\vec{x}(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\vec{x}(1) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, you're in a steady state.

$$\vec{x}_{steady} = Q \cdot \vec{x}_{steady}$$

$$Q\vec{x} = \vec{x} \longrightarrow Q\vec{x} - \vec{x} = \vec{0} \longrightarrow Q\vec{x} - I\vec{x} = \vec{0} \longrightarrow \underbrace{(Q-I)}_{\text{matrix}}\vec{x} = \vec{0}$$

$\vec{x}$ belongs to the Nullspace of $(Q-I)$
and it is an eigenvector, belonging to
the eigenspace corresponding to eigenvalue 1.