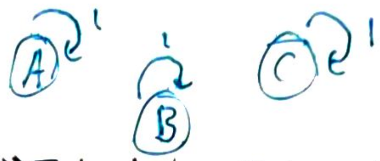
EECS 16A ## 9.17 LECTURE 6 ##

Matrix Inversion

ex. $Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\vec{x}(t+1) = Q\vec{x}(t)$



"Identity Matrix"

ex. $Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ $R = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\vec{x}(1) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

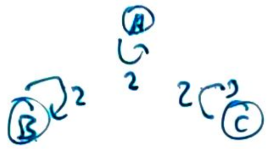
$\vec{x}(2) = Q\vec{x}(1) \longrightarrow \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$

$\vec{x}(3) = R\vec{x}(2) \longrightarrow \begin{bmatrix} 1/2 \\ 3/2 \\ 2 \end{bmatrix}$

$= R\cdot(Q\vec{x}(1)) = (R\cdot Q)\,\vec{x}(1)$

$$\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$$

ex. $Q = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$



If a pump leaks or generates, it is non-conservative.

$\vec{x}(t+1) = Q\cdot\vec{x}(t)$

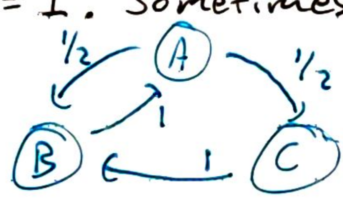
Recover $\vec{x}(t)$ from $\vec{x}(t+1)$

$R = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$

$\vec{x}(t+1) = \underbrace{QR}_{\text{I(dentity)}}\vec{x}(t+1) \longleftarrow \vec{x}(t) = R\vec{x}(t+1)$

DEF Matrix P is the inverse of matrix Q (both square) if $PQ = QP = I$. Sometimes, we will denote $P = Q^{-1}$!

ex. $Q = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1 \\ 1/2 & 0 & 0 \end{bmatrix}$



$\vec{x}(t+1) = Q(\vec{x}(t))$

Find P such that

$\vec{x}(t) = P(\vec{x}(t+1)) \rightarrow QP = I$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1 \\ 1/2 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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$Q \cdot \vec{P_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ some setup for $\vec{P_2}, \vec{P_3}$

Augmented Matrix: $\left[\begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & 0 & 0 \\ 1/2 & 0 & 1 & 0 & 1 & 0 \\ 1/2 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 & 0 & 2 \end{array}\right]$

$P = \begin{bmatrix} 0 & 0 & 2 \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix} \leftarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array}\right]$

THEOREM: If $QP = I$ and $RQ = I$, then $R = P$.
"Left inverse and right inverse are the same."

PROOF: $QP = RQ$ doesn't work

$R(QP) = R(I)$

$(RQ)P = R(I)$

$IP = R(I) \longrightarrow P = R$

$(QP)R = IR$

$QPR = R$

we didn't do this bc we know nothing about PR.

■ QED.

Not everything has an inverse: $Q = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \rightarrow$ cannot recover $\vec{x}(t)$ from $\vec{x}(t+1)$.

Inversion $\longleftrightarrow$ Gaussian Elimination

GE, Unique Solutions $\longleftrightarrow$ Linear (In)dependence

THEOREM: If the columns of A are linearly dependent, then matrix A is not invertible.

PROOF: $p \Rightarrow q$, not $q \Rightarrow$ not p (contrapositive)

if A <u>is</u> invertible, columns of A are linearly independent.

$c_1\vec{a}_1 + c_2\vec{a}_2 + \ldots c_n\vec{a}_n = 0$ where all c_i are not 0.

then, A^{-1} doesn't exist.

Let's pretend that A^{-1} <u>does</u> exist.

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$$A\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = 0, \quad \vec{c} \neq 0$$

$$A^{-1}(A\vec{c}) = A^{-1}\cdot 0$$
$$(A^{-1}\cdot A)\vec{c} = 0$$
$$I\cdot\vec{c} = 0$$
$$\vec{c} = 0$$

Here's a contradiction!

▨ QED..