EECS 16A ## 9.5 Lecture 3

Gaussian Elim.: Trickier Examples

$$2y + 3z = 2$$
$$x + y = 1$$

$$\left[\begin{array}{ccc|c} 0 & 2 & 3 & 2 \\ 1 & 1 & 0 & 1 \end{array}\right] \rightarrow \text{swap}(R_1, R_2)$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 2 & 3 & 2 \end{array}\right] \rightarrow R_2/2 \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 3/2 & 1 \end{array}\right]$$

y z

$$\rightarrow R_1 - R_2 \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -3/2 & 0 \\ 0 & 1 & 3/2 & 1 \end{array}\right] \begin{array}{l} \leftarrow \text{relate } x \,\&\, z \\ \leftarrow \text{relate } y \,\&\, z \end{array}$$

z = anything
$x = \frac{3}{2} z$
$y = 1 - \frac{3}{2} z$

identity matrix

I's are called pivots

row-reduced echelon form

row echelon form:
- all nonzero rows are above zero rows
- leading coefficient of nonzero row is to the right of the leading coeff. of the row above

- leading coefficient is 1
- each column with a leading 1 has a zero everywhere else

Example: 3 EQ, 2 VAR

$$x + y = 2$$
$$x - y = 1$$
$$2x - 2y = 2$$

$$\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 1 & -1 & 1 \\ 2 & -2 & 2 \end{array}\right] \rightarrow R_2 - R_1 \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & -2 & -1 \\ 2 & -2 & 2 \end{array}\right] \rightarrow R_2/-2$$

$$\rightarrow \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1 & 1/2 \\ 2 & -2 & 2 \end{array}\right] \rightarrow R_3 - 2R_1 \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1 & 1/2 \\ 0 & -4 & -2 \end{array}\right] \rightarrow R_3/-4$$

$$\rightarrow \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1 & 1/2 \\ 0 & 1 & 1/2 \end{array}\right] \rightarrow R_3 - R_2 \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1 & 1/2 \\ 0 & 0 & 0 \end{array}\right] \rightarrow R_1 - R_2$$

$$\rightarrow \left[\begin{array}{cc|c} 1 & 0 & 3/2 \\ 0 & 1 & 1/2 \\ 0 & 0 & 0 \end{array}\right] \begin{array}{l} \rightarrow x = 3/2 \\ \rightarrow y = 1/2 \\ \end{array}$$

MATRIX-VECTOR NOTATION

Matrix: $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ Augmented Matrix: $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 4 & 5 & 6 \end{array}\right]$

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & \cdot & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \\ a_{m1} & a_{m2} & \cdot & a_{mn} \end{bmatrix}}_{m \times n} \cdot \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

$A \cdot \vec{x} = \vec{b}$

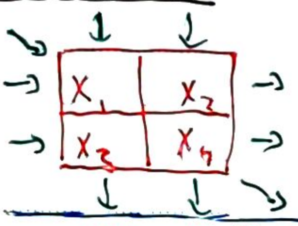
"operator"

matrix $\in \mathbb{R}^{m \times n}$ $\vec{x} \in \mathbb{R}^{n \times 1}$ $\vec{b} \in \mathbb{R}^{m \times 1}$

$$\begin{bmatrix} 1 & 2 & 3 \\ 9 & 8 & 7 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 + 2x_2 + 3x_3 \\ 9x_1 + 8x_2 + 7x_3 \end{bmatrix}$$

"row perspective on matrix-vector mult."

IMAGING LAB



M1: E1-4

$$\begin{array}{ll} 1 & x_1 + x_2 = b_1 \\ 2 & x_3 + x_4 = b_2 \\ 3 & x_1 + x_3 = b_3 \\ 4 & x_2 + x_4 = b_4 \\ 5 & x_1 + x_4 = b_5 \end{array}$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

M2: E1-3, 5

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_5 \end{bmatrix}$$

$A \vec{x} = \vec{b}$

engineer's choice (A) nature's choice ($\vec{b}$)

The goal is to understand just by looking at A whether we'll get a unique solution.

VECTOR PROPERTIES

1. Addition: $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $\vec{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ $\vec{x} + \vec{y} = \begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 \end{bmatrix}$
2. Scalar · Vector: $c\vec{x} = \begin{bmatrix} cx_1 \\ cx_2 \end{bmatrix}$

COLUMN VIEW ON MATRIX-VECTOR MULT.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} \vec{a}_1 & \vec{a}_2 & \vec{a}_3 \end{bmatrix} \qquad A\vec{x} = \begin{bmatrix} \vec{a}_1 & \vec{a}_2 & \vec{a}_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$\vec{a}_1 \quad \vec{a}_2 \quad \vec{a}_3$

$\vec{a}_1 x_1 + \vec{a}_2 x_2 + \vec{a}_3 x_3$ weights

"weighted sum of columns of A, linear-combin."

Does this give an interpretation for $A\vec{x} = \vec{b}$?
Can I express $\vec{b}$ as the linear combination of $\vec{a}_1, \vec{a}_2, \vec{a}_3$?

SPAN

Span of columns of matrix A is the set of all vectors $\vec{b}$ such that $A\vec{x} = \vec{b}$ has a solution, i.e. $\vec{b}$ is a linear combination of columns of A