EECS 16A ##9.3 LECTURE 2##

IMAGING

energy source → subject ← energy detector

BOSEA / DENEAO

$$M1 = \alpha_1 \cdot img_1 + \alpha_2 \cdot img_2$$
$$M2 = \beta_1 \cdot img_1 + \beta_2 \cdot img_2$$

known known ? known ?

each mix is a vector:

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad n = 512 \times 512$$

16A STRATEGY: SIMPLIFY

$$2x + 3y = 8 \quad (E_1)$$
$$3x - y = 1 \quad (E_2)$$

$2x + 3(3x-1) = 8$

$y = 3x - 1$ (insert more alg)

0. $\left[\begin{array}{cc|c} 2 & 3 & 8 \\ 3 & -1 & 1 \end{array}\right]$

Augmented Matrix

1. Set x coefficient to 1; "Normalize"

$E_1/2 = x + \frac{3}{2}y = 4 \quad (E_1*)$

2. Use E_1* to eliminate x from E_2

$E_2 - 3E_1* \quad 3x - y - 3x - \frac{9}{2}y = -11$

3. Solve for $y = 2$

1. $\left[\begin{array}{cc|c} 1 & 3/2 & 4 \\ 3 & -1 & 1 \end{array}\right]$

2. $\left[\begin{array}{cc|c} 1 & 3/2 & 4 \\ 0 & -11/2 & -11 \end{array}\right]$

3. $\left[\begin{array}{cc|c} 1 & 3/2 & 4 \\ 0 & 1 & 2 \end{array}\right]$

Gaussian Elimination!

ANOTHER REPRESENTATION

Matrix-vector Form: $\underbrace{\begin{bmatrix} 2 & 3 \\ 3 & -1 \end{bmatrix}}_{\text{matrix}} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{\text{vector}} = \underbrace{\begin{bmatrix} 8 \\ 1 \end{bmatrix}}_{\text{vector}}$

EECS 16A

Example 2

$2x + 3y = 8$
$2x + 3y = 6$

$$\left[\begin{array}{cc|c} 2 & 3 & 8 \\ 2 & 3 & 6 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 3/2 & 4 \\ 2 & 3 & 6 \end{array}\right] \rightarrow R_2 - 2R_1{}^{*}$$

$$\rightarrow \left[\begin{array}{cc|c} 1 & 3/2 & 4 \\ 0 & 0 & -2 \end{array}\right] \rightarrow 0x + 0y = -2$$

no solution!

Example 3

$x + 4y = 6$
$2x + 8y = 12$

$$\left[\begin{array}{cc|c} 1 & 4 & 6 \\ 2 & 8 & 12 \end{array}\right] \rightarrow R_2 - 2R_1 \rightarrow \left[\begin{array}{cc|c} 1 & 4 & 6 \\ 0 & 0 & 0 \end{array}\right]$$

infinite sols.! $0x + 0y = 0$

Gaussian Elimination

n variables, m equations

$$\alpha_{11} x_1 + \alpha_{12} x_2 + \dots + \alpha_{1n} x_n = \beta_1$$
$$\alpha_{21} x_1 + \alpha_{22} x_2 + \dots + \alpha_{2n} x_n = \beta_2$$
$$\underbrace{\alpha_{m1}}_{\text{known}} \underline{x_1} + \underline{\alpha_{m2}} \underbrace{x_2}_{\text{var}} + \dots + \underline{\alpha_{mn}} \underline{x_n} = \underline{\beta_m}$$

$$\left[\begin{array}{cccc|c} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} & \beta_1 \\ \alpha_{21} & \alpha_{22} & \cdot & \alpha_{2n} & \beta_2 \\ & & \vdots & & \vdots \\ \alpha_{m1} & \alpha_{m2} & \cdots & \alpha_{mn} & \beta_m \end{array}\right]$$

1. Start with row $i=1$
   * swap so x_i exists in row i.
   * coeff. of x_i in i should be 1.
     → multiply / divide as needed
2. For rows $j = [i+1, m]$, use row i to cancel x_i from j
3. Get upper triangular matrix
4. Back substitute

Stop Conditions

1. $[0\ 0\ \dots\ 0 \mid 0]$ infinite solutions
2. $[0\ \dots\ 0 \mid b]$ no solution $b \neq 0$
3. $[0\ \dots\ 0\ 1 \mid b]$ unique solution $b \neq 0$

EECS 16A

GEOMETRIC PERSPECTIVE

$2x + 3y = 8$

$3x - y = 1$