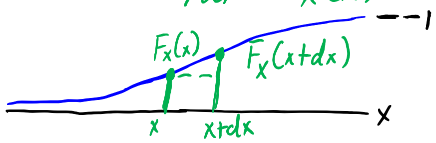
CS 70 ## 4.21 LECTURE 25

Probability Density Function

$$f_X(x) = \frac{F_X(x+dx) - F_X(x)}{dx} = \frac{d}{dx} F_X(x)$$

where $\underbrace{F_X(x)}_{CDF} = P_r(X \le x)$

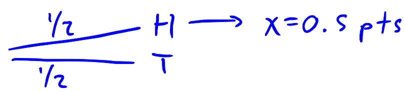
$\longrightarrow F_X(x) = \int_{-\infty}^{x} f_x(\tau)\,d\tau = P_r(X \le x)$

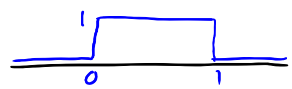


Takeaway: PDF is the derivative of CDF.

There is no limit on the magnitude of $f_X(x)$, so long as the CDF sums to 1.

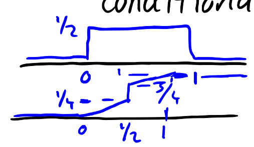
EX Hybrid RVs





$\underbrace{f_{X|T}(x)}_{\text{conditional PDF}}$

The probability of T is ½, so $f_{X|T}(x)$ would be $f_X(x)$
Doesn't add up to 1 though. Let's use the CDF:



Conditional PDFs

$f_{X|A}(x)$ $\qquad f_{X|A}(x)dx = P_r(x < X \le x+dx \mid A) = P_r(x < X \le x+dx \wedge X \in \mathcal{A})$

$\mathcal{A}$ is the set of values to which $X(\omega)$ maps whenever $\omega \in A$

$P(A) = P_r(X \in \mathcal{A})$ $\qquad$ Total probability: $f_X(x) = \sum_i f_{X|A_i}(x) P(A_i)$

EX Exponential RVs (continuous geometric)

$X \sim Exp(\lambda)$

$f_X(x) = \lambda e^{-\lambda x} \qquad x \ge 0$

$\int f_x(x)\,dx = \int_0^\infty \lambda e^{-\lambda x} dx = \lambda \int_0^\infty e^{-\lambda x}$

$= \lambda \cdot \frac{e^{-\lambda x}}{-\lambda}\Big|_0^\infty = 1 - e^{-\lambda x}\Big|_\infty = 1 \checkmark$

Expected Value

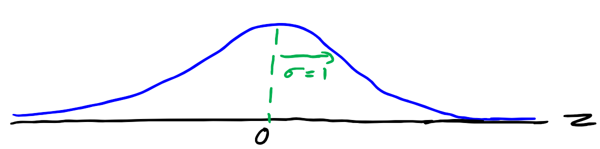
$E(X) = \int x f_X(x)\,dx \qquad E(X^2) = \int x^2 f_X(x)\,dx \qquad E[g(X)] = \int g(x) f_X(x)\,dx$

$\sigma_X^2 = E[(X - E(X))^2]$ $\qquad$ for an exponential RV, $E(X) = 1/\lambda \qquad \sigma_X^2 = 1/\lambda^2$

Gaussian/Normal RV

$Z \sim Norm(0,1) = Norm(\mu, \sigma)$

$f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$



$E(Z) = 0 \qquad \sigma_Z = \sigma_Z^2 = 1$

$\int_{-\infty}^{\infty} z f_Z(z)\,dz = 0 \qquad \sigma_Z^2 = E(Z^2) - E^2(Z) = E(Z^2) = 1 \qquad F_Z(z) = \Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\tau^2/2} d\tau$

no closed-form exp.!