CS 70 — ## 4.7 Lecture 21

Law of Expectation Invariance

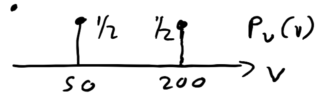
Let X, $P_X(x)$, $E(X)$, $Y=g(X)$. How do we find $E(Y)$?

$E(Y)=\sum_y y\,P_Y(y)$, but it's often hard to find $P_Y(y)$.

$\to E(Y)=E(g(X))=\sum_x g(x)\,P_X(x)$

If $g(x)=\alpha X+\beta$, then $E(g(x))=\alpha E(X)+\beta = g(E(X))$.

EX Boston $\xrightarrow{200 \text{ miles}}$ New York. $\frac{1}{2}$ airplane, 200 mph; $\frac{1}{2}$ car, 50 mph



Let's look at the expected travel time, where $T = d/v = 200/v$.

$T(v)$ isn't linear, so $E(T)=E\left(\frac{200}{v}\right) \neq \frac{200}{E(V)} = 1.6$ hours.

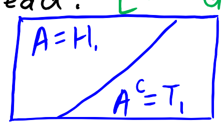
Correctly done, using LEI, yields $E\left(\frac{200}{v}\right)=\sum_v \frac{200}{v} P_V(v) = \frac{1}{2}+2 = 2.5$ hours.

Conditional RV

Let L be a RV. $P_L(\ell)=P_r(L=\ell)$. Then, $P_{L|A}(\ell)=\frac{P_r(L=\ell \wedge A)}{P(A)}$.

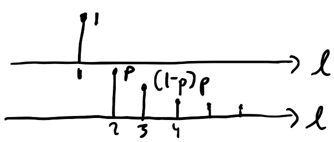
EX Flip a coin up to & including the first head. $L \sim Geom(p)$

$P_L(\ell)=(1-p)^{\ell-1}p \quad \forall \ell \geq 1$ Say there's



Condition on $P_{L|A}(\ell)$:

Condition on $P_{L|A^c}(\ell)$:



$\sum P_{L|A} = \sum P_{L|A^c} = 1$

$P_{L|A^c}(\ell) = P_L(\ell-1)$

you can think of this as one wasted flip.

$E(L|A)=\sum_\ell \ell\, P_{L|A}(\ell)$. What's $E(L|A)$? $E(L|A^c)$?

$\hookrightarrow 1$ $\qquad \hookrightarrow E(L)+1$

EV of Geom RV

$E(L)=1/p$. Think of a random binary sequence, where the distance between 1s is our 1st order interarrival of a random process, and is our EV. As p increases, EV naturally decreases. As p decreases, EV increases.

Law of Total Expectation

$$E(L)=\sum_\ell \ell P_L(\ell)=\sum_\ell \ell P_r(L=\ell)=\sum_\ell \ell \underbrace{\sum_{i=1}^{\infty} P_r(L=\ell|A_i)P(A_i)}_{=P_r(L=\ell)}$$

$= P_r(L=\ell) = B$, the conditioning event

$$= \sum_i P(A_i)\sum_\ell \ell\, P_{L|A_i}(\ell) = \sum_i P(A_i)E(L|A_i)$$