CS 70 ##4.2 LECTURE 20

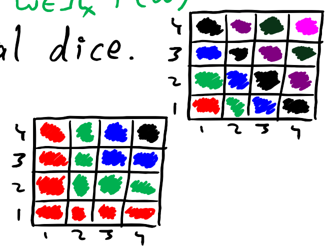
PMFs

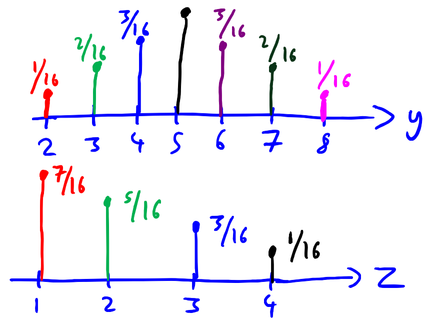
$P_X(x_i) = P(\Omega_X) = \sum_{\omega \in \Omega_X} P(\omega)$

EX Roll 2 tetrahedral dice.

$Y = X_1 + X_2$

$Z = \min(X_1 + X_2)$





most likely event is 5.

EXPECTED VALUE

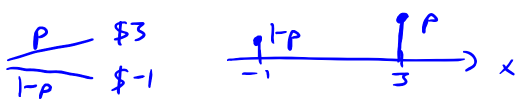
$E(X) = \sum_x x\, P_X(x) \longrightarrow$ weighted sum of values $x \in X$ with weights $P_X(x)$. this is a convex combination, which means the weights are non-negative and sum to 1.

EX Uniform PMF: $E(R) = \sum_{r=1}^{n} r P_R(r) = \frac{1}{n} \sum_{r=1}^{n} r = \frac{1}{n} \frac{n(n+1)}{2} = \frac{n+1}{2}$, center of mass

this has the same units as the random variable itself

This is the same thing as the mean of a random variable.

EX



here, we want $E(X) \geq 0$ or we'll likely lose money. $-1(1-p) + 3p \geq 0 \longrightarrow p \geq 1/4$.

if this is trade, you must be right at least 25% of the time (where "right" trades are good)

The 25% threshold here will break you even. Lower = loss, higher = profits.

BERNOULLI EV

$E(Z) = 0(1-p) + 1p = p$

BINOMIAL EV

$P_M(m) = \binom{n}{m}(1-p)^{n-m} p^m \longrightarrow E(M) = \sum_{m=0}^{n} m \binom{n}{m}(1-p)^{n-m} p^m$

Underlying Bernoulli: $\frac{p}{1-p}\,{}^{1}_{0}$ $\longrightarrow$ each result is $R_i \rightarrow M = \sum R_i$

$E(M) = E(R_1 + R_2 + \ldots + R_n) \longrightarrow$ $\boxed{\begin{array}{l} E(\alpha_1 X_1 + \alpha_2 X_2) = \alpha_1 E(X_1) + \alpha_2 E(X_2) \\ \text{Binomial Linearity of Expectation} \end{array}}$

$E(M) = E(R_1) + E(R_2) + \ldots + E(R_n) = p + p + \ldots + p = np$, which is nicer than

$E[g(X)] = g(E(X))$ iff $g(X) = \alpha X + \beta$, such as temperature conversion but not time comparisons when randomizing vehicles of different velocities.