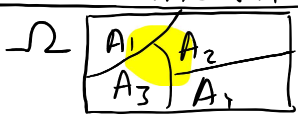
CS 70 ## 3.17 LECTURE 17

LAW OF TOTAL PROBABILITY



$\bigcup_{i=1}^{N} A_i = \Omega$

$A_i \bigcap_{i \neq j} A_j = \emptyset$

$P(B)$, by law of total prob., is $P(B \cap A_1) + P(B \cap A_2) + P(B \cap A_3) + P(B \cap A_4)$

$P(B) = \sum_{i=1}^{N} P(B \cap A_i)$ for N partitions of Ω.

UNIFORM PROBABILITY LAW

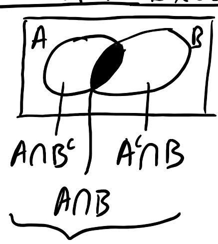
$P(G) = ?$ $\quad P(B) = ?$

$P(R) = ?$

$P(G)$, by uniform probability, is $|G|/|\Omega|$. $P(G) = 1/8$. $P(B) = 5/8$. $P(R) = 2/8$.

This works for finite sample spaces & where all sample points are equally likely (ex. each red ball is equally pickable).

INCLUSION-EXCLUSION PRINCIPLE



$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

if A & B are mutually exclusive, $P(A \cap B) = 0$.

↳ proof: $A \cap B = \{\omega \mid \omega \in A \wedge \omega \in B\}$. $\emptyset \cap \Omega = \emptyset \rightarrow$ axiom III

$\rightarrow P(\emptyset \cup \Omega) = P(\emptyset) + P(\Omega) = P(\emptyset) + 1$

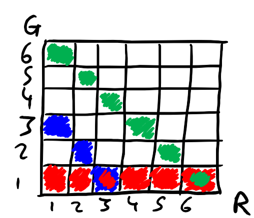
$\hookrightarrow = P(\Omega) = 1 = P(\emptyset) + 1 \rightarrow P(\emptyset) = 0$

$\hookrightarrow P(A \cup B) = P(A \cap B^c) + P(A \cap B) + P(A^c \cap B) + P(A \cap B) - P(A \cap B)$

$= \underbrace{P(A)}_{\text{law of total prob}} + \underbrace{P(B)}_{\text{law of total prob}} - P(A \cap B)$

CONDITIONAL PROBABILITY

Suppose we have a red 6-sided die & a green one.



$|\Omega| = 36$

B: sum 4

A: green 1

C: sum 7

$P(A|B) = ?$ $\quad P(A) = 1/6 \longrightarrow$ a priori probability

↳ probability that green die is 1, $\underbrace{\text{given sum is 4}}_{\text{conditioning event}}$.

Conditional sample space $\rightarrow$ B, size 3. We are now constrained here & must recalculate probabilities within this new space.

$P(A|B) = P(A \cap B)/P(B) = (|A \cap B|/|\Omega|)/(|B|/|\Omega|) = |A \cap B|/|B| = \frac{1/36}{3/36} = 1/3$

$P(C) = 1/6$. $P(A|C) = ? = \frac{1/36}{6/36} = 1/6$. In this case, $P(A|C) = P(A)$. Bug or feature?

A & C are independent; knowing 1 doesn't change likelihood of other.

Mutually exclusive events are as dependent as it gets.

BAYES'S RULE

$P(A|B) = P(A \cap B)/P(B)$ $\qquad$ $P(B|A) = P(A \cap B)/P(A)$

$P(A \cap B) = P(A|B)\,P(B)$ $\qquad$ $P(A \cap B) = P(B|A)\,P(A)$

$$P(A|B)\,P(B) = P(B|A)\,P(A)$$

$$\left.P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}\right\}\ \text{Bayes's Rule}$$