CS 70 ## 3.12 Lecture 16

It's Babak! Welcome to the world of randomness.
Fun read: *The Drunkard's Walk* by Leonard Mlodinow.

Random Experiment

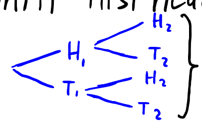
The set of possible outcomes is known, but the outcome of a specific trial is not.

DEF *Sample Space*: finest grain set of mutually exclusive & collectively exhaustive outcomes.

EX Roll a 6-sided die. $\Omega = \{1, 2, 3, 4, 5, 6\}$ (sample points)

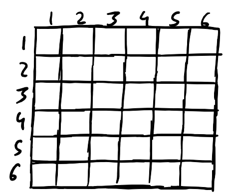
EX Flip a coin until first head. $\Omega = \{H_1, T_1H_2, T_1T_2H_3, T_1T_2T_3H_4, \ldots\}$

EX Flip 2 coins.



tree diagram

EX Roll 2 6-sided dice. →



draw, baby, draw!

DEF *Event*: any subset $A \in \Omega$

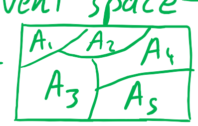
EX Roll 2 6-sided dice. A: 1^{st} toss even; $A = \{(2,1), (2,2), \ldots, (2,6), (4,1), \ldots, (4,6), (6,1), \ldots (6,6)\}$

Suppose $|\Omega| = n$. How many events can be defined on Ω? 2^n. For each of n sample points, you either include it or not. Binary choice! (or, $\sum_{k=0}^{n} \binom{n}{k} = 2^n$)

Probability

A mapping P: event space $\longrightarrow \mathbb{R}$

DEF *Event Space*: Ω



$A_k \cap A_\ell = \emptyset$ if $k \neq \ell$

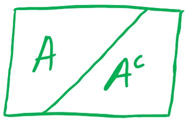
$\bigcup_{k=1}^{5} A_k = \Omega$

Axioms of Probability

I. *Nonnegativity*: for any event A, $P(A) \geq 0$.

II. *Normalization*: $P(\Omega) = 1$

III. *Additivity*: if $A_1 \cap A_2 = \emptyset$ (mutually exclusive), $P(A_1 \cup A_2) = P(A_1) + P(A_2)$

EX Ω 

A^c: complement of A

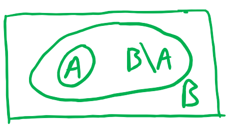
$A \cup A^c = \Omega \qquad A \cap A^c = \emptyset$

how can we relate $P(A)$ & $P(A^c)$?

$P(A \cup A^c) = P(A) + P(A^c) = P(\Omega) = 1 - P(A) = P(A^c)$

EX Flip a coin 1000 times. A: at least one flip is a head. $P(A) = ?$ $\quad P(A^c) =$ no heads $= \frac{1}{2^{1000}}$

$A \cap A^c = \emptyset$ & $A \cup A^c = \Omega \xrightarrow[\text{axioms of prob.}]{\text{using ex. above}} P(A) = 1 - P(A^c) = 1 - \frac{1}{2^{1000}}$

EX Ω 

$A \subseteq B$

$A \cap (B \setminus A) = \emptyset,\ A \cup (B \setminus A) = B \longrightarrow P(A) = P(B) - \underbrace{P(B \setminus A)}_{>0}$

$\longrightarrow P(A) \leq P(B) \longrightarrow$ monatonicity property