# CS 70 ## 2.6 LECTURE 6 ##

## GRAPHS

$G(V, E)$ $E = \{(1,2), (1,3), (1,4), (2,4), (3,4)\}$
$V = \{1,2,3,4\}$

Directed Graphs have directional arrows.
$u$ is a neighbor of $v$ if $\{u,v\} \in E$.
$\{u,v\}$ is incident to $u$ & $v$
Degree of $u$ is number of incident edges
Sum of degrees is twice the number of edges right?
Edges are incident to vertices. Each edge has two incidents. Proved by definition.
Paths are connected sequences of edges. $k$ V, $k-1$ E
Cycles are paths that start & end @ same V. $k-1$ V, $k-1$ E
Walks can have repeated vertices/edges.
Tours are cyclic walks.

## TREES

Connected graph without a cycle.
Connected graph with $|V|-1$ edges.

**THM** The statements above are equivalent.

**LEMMA** If $v$ is degree 1 in connected graph $G$, $G-v$ is connected

**PROOF** For $x \neq v, y \neq v \in V$, there is a path between them w/o $v$, as $v$ is degree 1. So, $G-v$ is connected. By induction on $|V|$

**BASE** $|V|=1$, $0=|V|-1$ edges, no cycles.

**CLAIM** There is a degree 1 node.

**PROOF** Every vertex degree $\geq 1$. $\Sigma$ degrees $= 2|V|-2$. Avg $= 2 - 2/|V|$
Not everyone can be bigger than average. $\square$
$G-v$ has $|V|-1$ vertices & $|V|-2$ edges, so by induction there's no cycle in $G-v$. $v$ is degree 1, so no cycles in $G$.

Proof of only if is above. Proof of if was not copied.
In a connected planar graph, $v+f=e+2$ Euler's Formula!