CS 70 ## 1.23 LECTURE 2

NEGATION

$\neg(A \land B) \equiv \neg A \lor \neg B$ $\qquad \neg(\forall x\ P(x)) \equiv \exists x\ \neg P(x)$

$\neg(A \lor B) \equiv \neg A \land \neg B$ $\qquad \neg(\exists x\ P(x)) \equiv \forall x\ \neg P(x)$

PROOFS

DIRECT

Straight shot $\qquad P \Rightarrow Q$

Contraposition $\qquad \neg Q \Rightarrow \neg P$

CONTRADICTION

CASES

INDUCTION

ex. See n3 for examples. Let's build on THM 3.2.

THM If sum of digits of n divisible by 9, n divisible by 9.

PROOF $n = \sum_{l=0}^{m} 10^l d[l]$ $\qquad s = \sum_{l=0}^{m} d[l]$ $\qquad$ (let)

$\text{nines}(l) = \underbrace{99\ \ 9}_{l \text{ times}}$ $\qquad \text{ones}(l) = \underbrace{11\ \ 1}_{l \text{ times}}$ $\qquad$ (definition)

$10^l = \text{nines}(l) + 1$ $\qquad 9 \cdot \text{ones}(l) = \text{nines}(l)$ $\qquad$ (obvious)

START $s = 9k$ $\qquad s + \sum_{l=0}^{m} \text{nines}(l)\, d[l] = 9k + 9 \sum_{l=0}^{m} \text{ones}(l)\, d[l]$

$\sum_{l=0}^{m} (1 + \text{nines}(l))\, d[l] = 9(k + \Sigma)$ $\qquad n = 9(k + \Sigma)$ $\qquad$ ▨

assumed! must prove geometric sums, or claim obvious

ex. Pigeonhole Principle

THM $n, k \in \mathbb{Z}^+$; n pigeons, k holes. If $n > k$, then at least one hole will have more than 1 pigeon.

PROOF All holes have at most 1 pigeon $\qquad$ (contraposition)

Thus, $n \le k$. Untrue. $\qquad$ ▨

$n = \sum_{i=1}^{k} p[i]$ $\qquad n \le \sum_{i=1}^{k} 1 = k$ $\qquad$ (let)